Resultant of Concurrent Force Sys

Resultant of a force system is a force or a couple that will have
to the body, both in translation and rotation, if all the forces ar
replaced by the resultant.
The equation involving the resultant of force system are the fol

1. $R_x = \Sigma F_x = F_{x1} + F_{x2} + F_{x3} + \ldots$

   The x-component of the resultant is equal to the summatio
   the x-direction.

2. $R_y = \Sigma F_y$:

   The y-component of the resultant is equal to the summatio
   the y-direction.

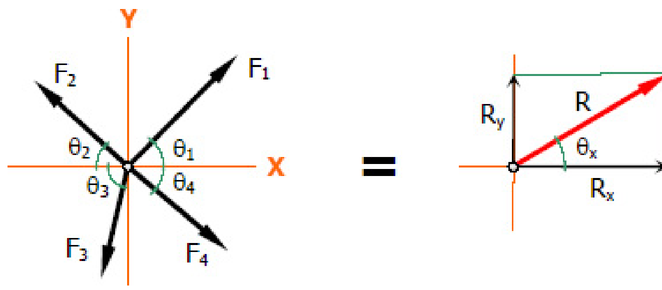
3. $R_z = \Sigma F_z$:

   The z-component of the resultant is equal to the summatio
   the z-direction.

Note that according to the type of force system, one or two or
equations above will be used in finding the resultant.

Resultant of Coplanar Concurrent Force System

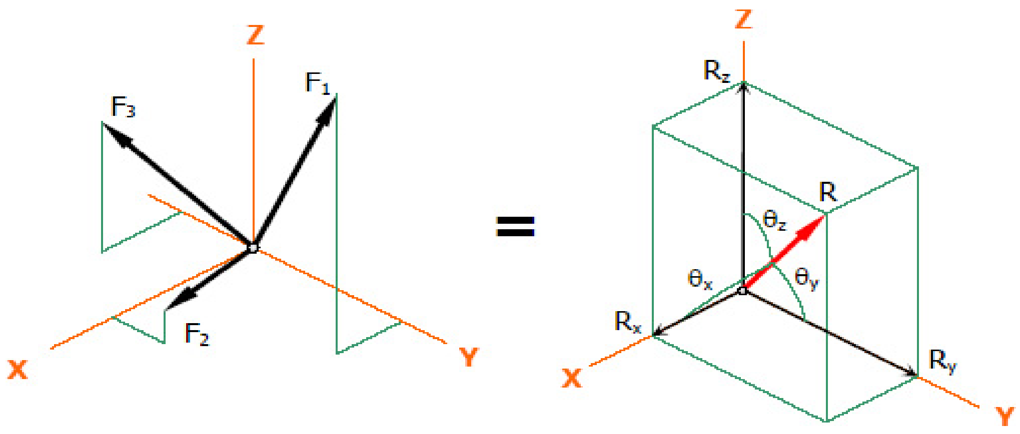
The line of action of each forces in coplanar concurrent force system a
plane. All of these forces meet at a common point, thus concurrent. In
resultant can be found by the following formulas:



$$R_x = \Sigma F_x$$
$$R_y = \Sigma F_y$$
$$R = \sqrt{{R_x}^2 + {R_y}^2}$$
$$\tan\theta_x = \frac{R_y}{R_x}$$

Resultant of Spatial Concurrent Force System

Spatial concurrent forces (forces in 3-dimensional space) meet at a common point but do not lie in a single plane. The resultant can be found as follows:



$$R_x = \Sigma F_x$$
$$R_y = \Sigma F_y$$
$$R_z = \Sigma F_z$$
$$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$$

Direction Cosines

$$\cos\theta_x = \frac{R_x}{R}$$
$$\cos\theta_y = \frac{R_y}{R}$$
$$\cos\theta_z = \frac{R_z}{R}$$

Vector Notation of the Resultant

$\mathbf{R} = \Sigma\mathbf{F}$

$\mathbf{R} = (\Sigma F_x)\mathbf{i} + (\Sigma F_y)\mathbf{j} + (\Sigma F_z)\mathbf{k}$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} + R_z\mathbf{k}$$

Where

$R_x = \Sigma F_x$

$R_y = \Sigma F_y$

$R_z = \Sigma F_z$

$R = \sqrt{{R_x}^2 + {R_y}^2 + {R_z}^2}$

Problem 011

Three ropes are tied to a small metal ring. At the end of each rope three students are pulling, each trying to move the ring in their direction. If we look down from above, the forces and directions they are applying are shown in Fig. P-011. Find the net force on the ring due to the three applied forces.

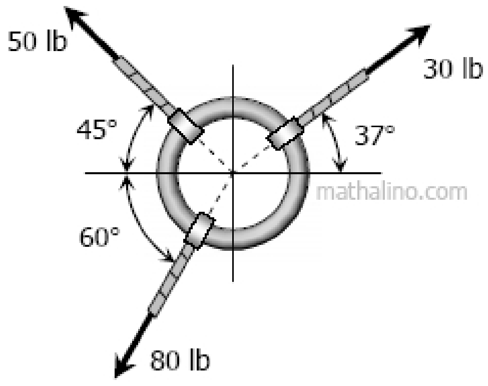
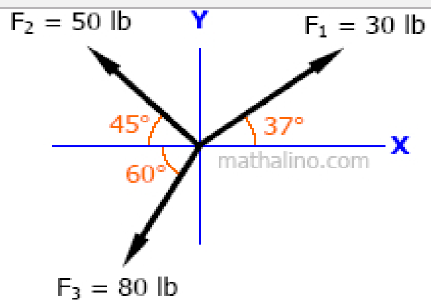


Figure 011

Solution 011



$R_x = \Sigma F_x$

$R_x = 30 \cos 37° - 50 \cos 45° - 80 \cos 60°$

$R_x = -51.40 \text{ lb}$

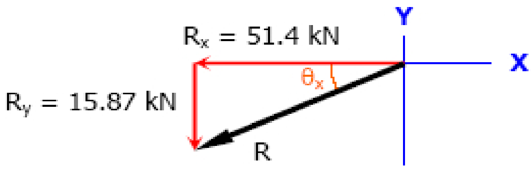
$R_x = 51.40 \text{ lb to the left}$

$R_y = \Sigma F_y$

$R_y = 30 \sin 37° + 50 \sin 45° - 80 \sin 60°$

$R_y = -15.87 \text{ lb}$

$R_y = 15.87 \text{ lb downward}$



$R = \sqrt{R_x{}^2 + R_y{}^2}$

$R = \sqrt{51.40^2 + 15.87^2}$

$R = 53.79 \text{ lb}$

$\tan \theta_x = \dfrac{R_y}{R_x} = \dfrac{15.87}{51.40}$

$\theta_x = 17.16°$

Thus, the net force on the ring is 53.79 lb downward to the left at $\theta_x = 17.16°$.

Problem 012

Find the resultant vector of vectors **A** and **B** shown in Fig. P-012.

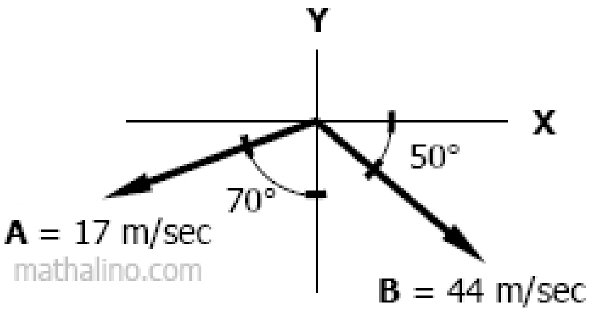


Figure P-012

Solution 012: Component Method

$R_x = \Sigma F_x$

$R_x = 44\cos 50° - 17\sin 70°$

$R_x = 12.31 \text{ m/sec to the right}$

$R_y = \Sigma F_y$

$R_y = -44\sin 50° - 17\cos 70°$

$R_y = -27.89 \text{ m/sec}$

$R_y = 39.52 \text{ m/sec downward}$

$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{12.31^2 + 39.52^2}$

$R = 41.39 \text{ m/sec}$

$\tan\theta_x = \dfrac{R_y}{R_x} = \dfrac{39.52}{12.31}$

$\theta_x = 72.70°$

The resultant vector R = 41.39 m/sec downward to the right at $\theta_x = 72.70°$.

Problem 014

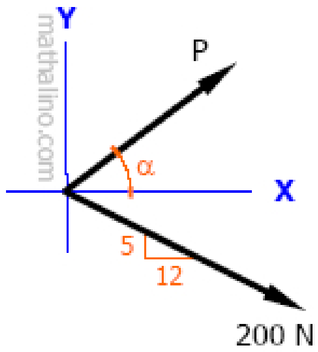


Figure P-014

From Fig. P-014, P is directed at an angle α from x-axis and the 200 N force is acting at a slope of 5 vertical to 12 horizontal.

1. Find P and α if the resultant is 500 N to the right along the x-axis.
2. Find P and α if the resultant is 500 N upward to the right with a slope of 3 horizontal to 4 vertical.
3. Find P and α if the resultant is zero.

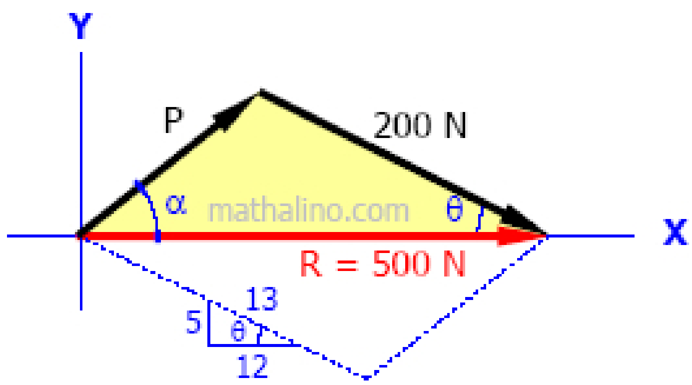
Part a: The resultant is 500N to the right along the x-axis

By Cosine law of the shaded triangle

$$P^2 = 200^2 + 500^2 - 2(200)(500)\cos\theta$$

$$P^2 = 200^2 + 500^2 - 2(200)(500)(\tfrac{12}{13})$$

$$P = 324.63\ \text{N} \qquad \textit{answer}$$



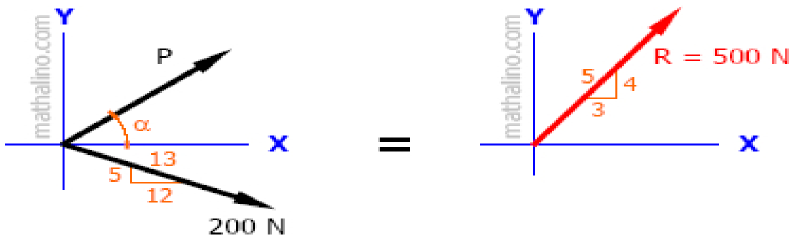
By Sine law

$$\frac{P}{\sin\theta} = \frac{200}{\sin\alpha}$$

$$\sin\alpha = \frac{200\sin\theta}{P} = \frac{200(\frac{5}{13})}{324.63}$$

$$\alpha = 13.71^\circ \qquad \textit{answer}$$

Part b: The resultant is 500 N upward to the right with a slope of 3 horizontal to 4 vertical



$$R_x = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$R_y = 500(\tfrac{4}{5}) = 400 \text{ N}$$

$$R_x = P\cos\alpha + 200(\tfrac{12}{13})$$

$$300 = P\cos\alpha + 184.61$$

$$P\cos\alpha = 115.39$$

$$P = \frac{115.39}{\cos\alpha}$$

$$R_y = P\sin\alpha - 200(\tfrac{5}{13})$$

$$400 = P\sin\alpha - 76.92$$

$$P\sin\alpha = 476.92$$

$$\left(\frac{115.39}{\cos\alpha}\right)\sin\alpha = 476.92$$

$$115.38\tan\alpha = 476.92$$

$$\tan\alpha = 4.1335$$

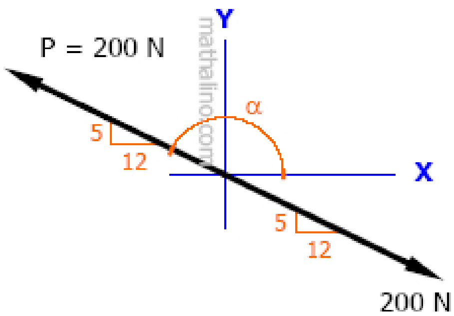
$$\alpha = 76.4^\circ \qquad \textit{answer}$$

$$P = \frac{115.39}{\cos 76.4^\circ}$$

$$P = 490.68 \text{ N} \qquad \textit{answer}$$

Part c: The resultant is zero

The resultant is zero if P and the 200 N force are equal in magnitude, oppositely directed, and collinear.



$\alpha = 180^\circ - \arctan\frac{5}{12}$

$\alpha = 157.38^\circ$

Thus, P = 200 N at $\alpha = 157.38^\circ$ *answer*

Problem 015

Forces F, P, and T are concurrent and acting in the direction as shown in Fig. P-015.

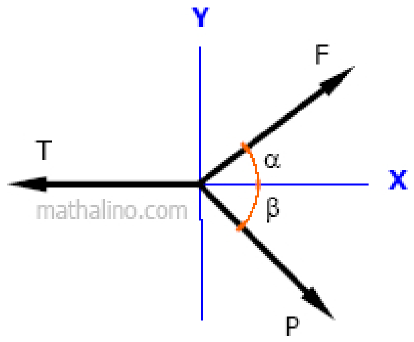
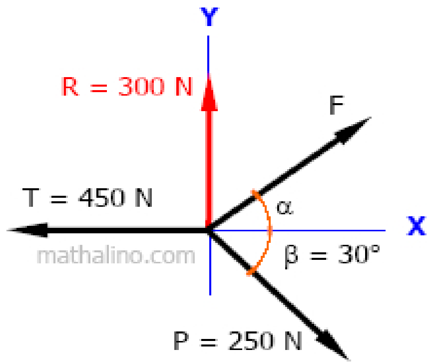


Figure P-015

1. Find the value of F and α if T = 450 N, P = 250 N, β = 30Â°, and the resultant is 300 N acting up along the y-axis.
2. Find the value of F and α if T = 450 N, P = 250 N, β = 30Â° and the resultant is zero.
3. Find the value of α and β if T = 450 N, P = 250 N, F = 350 N, and the resultant is zero.

Part a: Unknown force and direction with non-zero resultant

$R_x = 0$ and $R_y = 300\ \text{N}$



$R_x = \Sigma F_x$

$0 = F\cos\alpha + 250\cos 30^\circ - 450$

$F\cos\alpha = 233.49$

$F = \dfrac{233.49}{\cos\alpha}$

$R_y = \Sigma F_y$

$300 = F\sin\alpha - 250\sin 30^\circ$

$F\sin\alpha = 425$

$\left(\dfrac{233.49}{\cos\alpha}\right)\sin\alpha = 425$

$\tan\alpha = 1.8202$

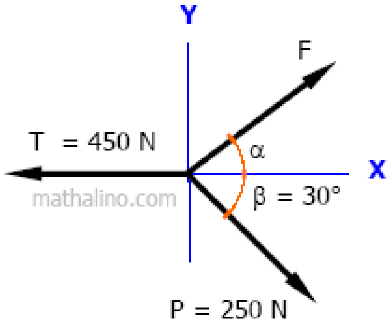
$\alpha = 61.22^\circ \qquad \textit{answer}$

$F = \dfrac{233.49}{\cos 61.22^\circ}$

$F = 484.92\ \text{N} \qquad \textit{answer}$

Part b: Unknown force and direction with zero resultant

$R_x = 0$ and $R_y = 0$



$R_x = \Sigma F_x$

$0 = F \cos \alpha + 250 \cos 30^\circ - 450$

$F \cos \alpha = 233.49$

$F = \dfrac{233.49}{\cos \alpha}$

$R_y = \Sigma F_y$

$0 = F \sin \alpha - 250 \sin 30^\circ$

$F \sin \alpha = 125$

$\left(\dfrac{233.49}{\cos \alpha}\right) \sin \alpha = 125$

$\tan \alpha = 0.5354$

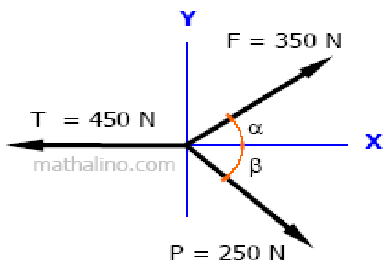
$\alpha = 28.16^\circ$ *answer*

$F = \dfrac{233.49}{\cos 28.16^\circ}$

$F = 264.85 \text{ N}$ *answer*

Part c: Unknown direction of two forces with zero resultant

$R_x = 0$ and $R_y = 0$



$R_y = \Sigma F_y$

$0 = 350 \sin \alpha - 250 \sin \beta$

$7 \sin \alpha - 5 \sin \alpha = 0$

$7 \sin \alpha = 5 \sin \beta$

$49 \sin^2 \alpha = 25 \sin^2 \beta$ $\rightarrow$ Equation (1)

$R_x = \Sigma F_x$

$0 = 350 \cos \alpha + 250 \cos \beta - 450$

$7 \cos \alpha + 5 \cos \beta - 9 = 0$

$7 \cos \alpha = 9 - 5 \cos \alpha$

$49 \cos^2 \alpha = (9 - 5 \cos \alpha)^2$

$49 \cos^2 \alpha = 81 - 90 \cos \beta + 25 \cos^2 \beta$ $\rightarrow$ Equation (2)

Equation (1) + Equation (2)

$49 \sin^2 \alpha + 49 \cos^2 \alpha = 25 \sin^2 \beta + (81 - 90 \cos \beta + 25 \cos^2 \beta)$

$49(\sin^2 \alpha + \cos^2 \alpha) = 25(\sin^2 \beta + \cos^2 \beta) + 81 - 90 \cos \beta$

$49(1) = 25(1) + 81 - 90 \cos \beta$

$90 \cos \beta = 25 + 81 - 49$

$\cos \beta = \frac{57}{90}$

$\beta = 50.70^\circ$ *answer*

From Equation (1)

$49 \sin^2 \alpha = 25 \sin^2 50.70^\circ$

$7 \sin \alpha = 5 \sin 50.70^\circ$

$\sin \alpha = \frac{5}{7} \sin 50.70^\circ$

$\alpha = 33.56^\circ$ *answer*

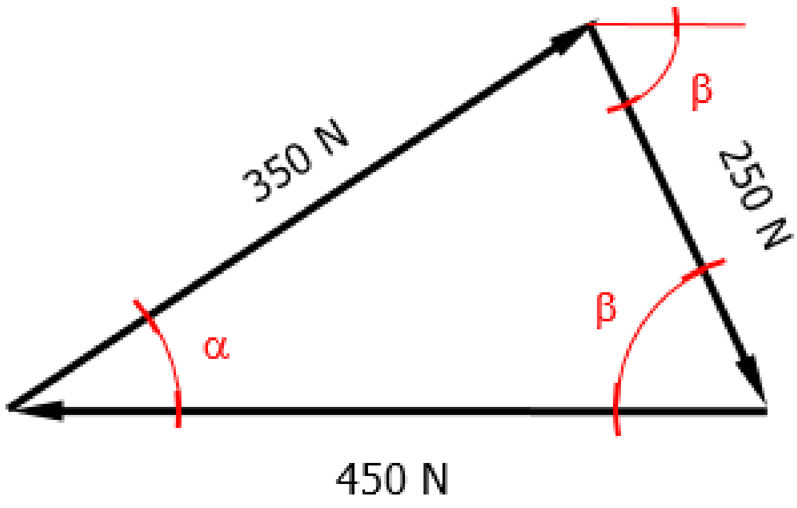
Another Solution for Part c

By Cosine Law

$250^2 = 350^2 + 450^2 - 2(350)(450)\cos\alpha$

$$\cos\alpha = \frac{350^2 + 450^2 - 250^2}{2(350)(450)}$$

$\alpha = 33.557^\circ \qquad answer$

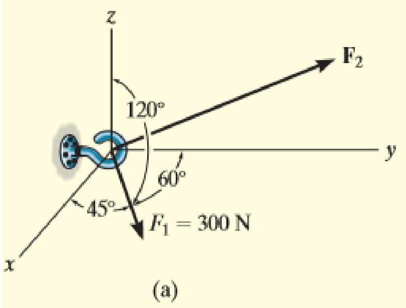


$350^2 = 250^2 + 450^2 - 2(250)(450)\cos\beta$

$$\cos\beta = \frac{250^2 + 450^2 - 350^2}{2(250)(450)}$$

$\beta = 50.704^\circ \qquad answer$

Example:



Two forces act on the hook shown in Fig. 2–33*a*. Specify the magnitude of $\mathbf{F}_2$ and its coordinate direction angles so that the resultant force $\mathbf{F}_R$ acts along the positive y axis and has a magnitude of 800 N.

SOLUTION

To solve this problem, the resultant force $\mathbf{F}_R$ and its two components, $\mathbf{F}_1$ and $\mathbf{F}_2$, will each be expressed in Cartesian vector form. Then, as shown in Fig. 2–33*b*, it is necessary that $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$.

Applying Eq. 2–9,

$$\begin{aligned}
\mathbf{F}_1 &= F_1 \cos\alpha_1 \mathbf{i} + F_1 \cos\beta_1 \mathbf{j} + F_1 \cos\gamma_1 \mathbf{k} \\
&= 300 \cos 45^\circ\, \mathbf{i} + 300 \cos 60^\circ\, \mathbf{j} + 300 \cos 120^\circ\, \mathbf{k} \\
&= \{212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k}\}\,\text{N} \\
\mathbf{F}_2 &= F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k}
\end{aligned}$$

Since $\mathbf{F}_R$ has a magnitude of 800 N and acts in the $+\mathbf{j}$ direction,

$$\mathbf{F}_R = (800\text{ N})(+\mathbf{j}) = \{800\mathbf{j}\}\text{ N}$$

We require

$$\begin{aligned}
\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 \\
800\mathbf{j} &= 212.1\mathbf{i} + 150\mathbf{j} - 150\mathbf{k} + F_{2x}\mathbf{i} + F_{2y}\mathbf{j} + F_{2z}\mathbf{k} \\
800\mathbf{j} &= (212.1 + F_{2x})\mathbf{i} + (150 + F_{2y})\mathbf{j} + (-150 + F_{2z})\mathbf{k}
\end{aligned}$$

To satisfy this equation the $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components of $\mathbf{F}_R$ must be equal to the corresponding $\mathbf{i}, \mathbf{j}, \mathbf{k}$ components of $(\mathbf{F}_1 + \mathbf{F}_2)$. Hence,

$$\begin{aligned}
0 &= 212.1 + F_{2x} & F_{2x} &= -212.1\text{ N} \\
800 &= 150 + F_{2y} & F_{2y} &= 650\text{ N} \\
0 &= -150 + F_{2z} & F_{2z} &= 150\text{ N}
\end{aligned}$$

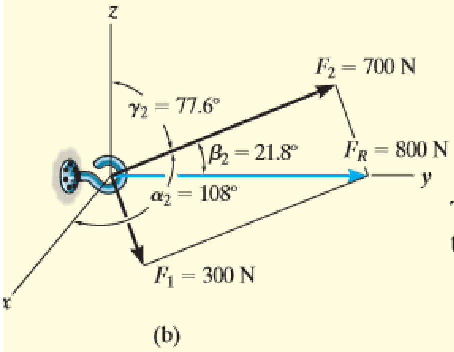


Fig. 2–33

The magnitude of $\mathbf{F}_2$ is thus

$$\begin{aligned}
F_2 &= \sqrt{(-212.1\text{ N})^2 + (650\text{ N})^2 + (150\text{ N})^2} \\
&= 700\text{ N} && \textit{Ans.}
\end{aligned}$$

We can use Eq. 2–9 to determine $\alpha_2, \beta_2, \gamma_2$.

$$\cos\alpha_2 = \frac{-212.1}{700}; \qquad \alpha_2 = 108^\circ \qquad \textit{Ans.}$$

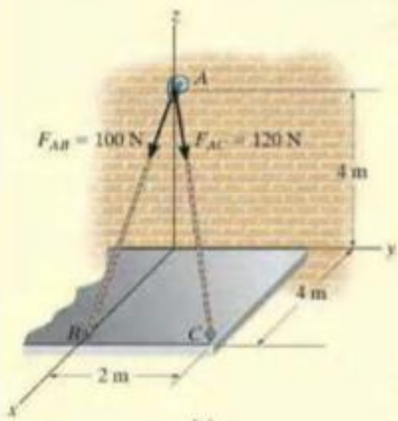
$$\cos\beta_2 = \frac{650}{700}; \qquad \beta_2 = 21.8^\circ \qquad \textit{Ans.}$$

$$\cos\gamma_2 = \frac{150}{700}; \qquad \gamma_2 = 77.6^\circ \qquad \textit{Ans.}$$

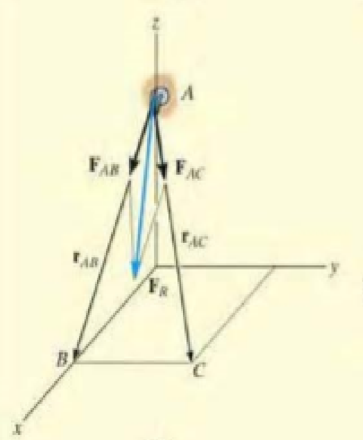
These results are shown in Fig. 2–33*b*.

Example 14: The roof is supported by cables as shown in the photo. If the cables exert forces F_{AB}= 100 N and F_{AC}= 120 N on the wall hook at A as shown in Figure (a), determine the resultant force acting at A. Express the result as Cartesian components.

Solution: Find the Cartesian projections of the r:



(a)



(b)

For F_{AB}:

$$\Delta x_1 = x_2 - x_1 = 4 - 0 = 4\,\text{m}$$

$$\Delta y_1 = y_2 - y_1 = 0 - 0 = 0\,\text{m}$$

$$\Delta z_1 = z_2 - z_1 = 0 - 4 = -4\,\text{m}$$

$$r_{AB} = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} = \sqrt{4^2 + 0^2 + (-4)^2} = 5.65\,\text{m}$$

For F_{AC}:

$$\Delta x_2 = x_2 - x_1 = 4 - 0 = 4\,\text{m}$$

$$\Delta y_2 = y_2 - y_1 = 2 - 0 = 2\,\text{m}$$

$$\Delta z_2 = z_2 - z_1 = 0 - 4 = -4\,\text{m}$$

$$r_{AC} = \sqrt{(\Delta x_2)^2 + (\Delta y_2)^2 + (\Delta z_2)^2} = \sqrt{4^2 + 2^2 + (-4)^2} = 6.0\,\text{m}$$

The components of resultant force:

$$F_x = \sum F_{ix} = \frac{\Delta x_1}{r_{AB}} F_{AB} + \frac{\Delta x_2}{r_{AC}} F_{AC} = \frac{4}{5.66} * 100 + \frac{4}{6} * 120 = 151\,\text{N}$$

$$F_y = \sum F_{iy} = \frac{\Delta y_1}{r_{AB}} F_{AB} + \frac{\Delta y_2}{r_{AB}} F_{AC} = \frac{0}{5.66} * 100 + \frac{2}{6} * 120 = 40\,\text{N}$$

$$F_z = \sum F_{ix} = \frac{\Delta z_1}{r_{AB}} F_{AB} + \frac{\Delta z_2}{r_{AC}} F_{AC} = \frac{(-4)}{5.66} * 100 + \frac{(-4)}{6} * 120 = -151\,\text{N}$$

And the resultant

$$F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{151^2 + 40^2 + (-151)^2} = 216\,\text{N}$$